2017 13th International Conference on Natural Computation, Fuzzy Systems and Knowledge Discovery (ICNC-FSKD 2017)

Rough Membership Measure in Intuitionistic Fuzzy Information System

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***Abstract*—Fuzzy relation is the generalization of the classical binary relation. Based on intuitionistic fuzzy information system, a fuzzy equivalence relation is defined in this paper. Thus, fuzzy approximation space is established in intuitionistic fuzzy information system. And rough membership measure is defined by fuzzy equivalence relation. In addition, a few significant properties of the degree of the roughness of membership are proved. , after that an demonstrative case is proposed in intuitionistic fuzzy information system.**

***Index Terms*—Fuzzy equivalence relation; IF information system; Rough membership measure; Rough set**

I. Introduction

In 1980, Pawlak [39] proposed the rough set theory, which is an expansion of the classical set theory. And in the field of artificial intelligence[14], rough sets theory has a wide range of applications. And rough set theory be considered as a soft computing tool to handle inaccurate data sets. Rough set theory has been used adequately in the domains of pattern recognition[30][16], medical diagnosis[41][2], algebra[18][11], data mining[42][28], conflict analysis [29][7][31], which is related to a large amount of inaccurate, fuzzy, and undeterminate information.

In 1986 intuitionistic fuzzy (IF) set was proposed by Atanassov [15]. As a extension of fuzzy set [19], intuitionistic fuzzy is highly effective to deal with vagueness[27]. IF set theory has been widely used in deal with inaccurate of data [12][21]. The combination of IF set theory and rough set theory can produce a new hybrid mathematical structure to deal with IF data.[13]. For the near term, the definition of IF sets has been discussed in the IF information system, and the rough set theory has been extended[8][1]. For example Wu and Liu [32] discussed IF equivalence in IF information system, and founded upper approximation reduction model in IF information system. Huang et al.[3] proposed three types of IF multi-granulation rough set (IFMGRS, briefly). From their basic properties, Huang et al. concluded that they are generalizes of above of three IF rough sets(IFRS). And, Huang et al. have defined the reduce of the three types of IFMGRSs to remove superfluous IF granulations. IFRS approximation method is proposed by Zhou et al.[22]. And Zhou has improved algorithm about IFRS approximation method in[23]. Zhang[43] studied some properties and conclusions of upper and lower approximation of IF covering. Huang et al. [3][6][4][5] have proposed interval-valued IF and order-based IF rough set models and their applications. Rough membership function was proposed by Pawlak [40] in 1994. These functions have values are not $\{0,1\}$, but interval [0,1]. Rough membership function is computable in the light of the observable information about the objects rather than on the objects themselves. Greco et al. [10] applied Rough membership function to Bayesian decision theory. A new decision theoretic model is constructed by combining decision theory with rough membership. Mani [24] applied Rough membership function to probability. A probabilistic rough membership measurement model is constructed by combining probability with rough membership. Li and Xu [33] discussed rough membership degrees in ordered information system. Ge et al.[36] researched what to utilize Pawlak's rough membership function on digitally featuring decisions by use an case study in evidence-based medicine. Yao [38] proposed probabilistic rough set approximation operators by use rough membership functions. Based on existing studies , he gave some valuable conclusions about the rough set model with decision-theoretic. Through the use of the idea of rough membership functions, the category theoretic approach was generalized by Chakraborty[25]. Based on a new covering-based rough membership function, Xu [35]studied generalized upper approximation and lower approximation. They also defined a measure of roughness on account of the covering-based rough membership function and discussed some significant applications of this measure. However, based on IF information system, how to research rough membership measure. The problem remains to be studied. Thus, based on IF information system, we structure fuzzy approximation space by a fuzzy equivalence relation. And rough membership measure is redefined in fuzzy approximation space. after that, a few significant properties of the degree of the roughness of membership are proved.

The remaining of this article is shown below: a few preliminary definitions are concisely showed in Section 2. Fuzzy approximation space is obtained by defined the fuzzy equivalence relation in IF information system .Rough membership measure is defined by fuzzy equivalence relation. Therefore, a few significant properties of rough memberships measure have been presented and demonstrated in Section 3. In Section 4, these properties are validated with an examples.

II. PRELIMINARIES

A. Fuzzy Set, IF Set and IF Information System(IFIS)

Definition 2.1[20][**?**]Suppose U be a universe of discourse

$$A : U \to [0,1]; u| \to A(x)$$

then A is called fuzzy set on U

(1) $\forall x \in U$, $B(x) \leq A(x) \Rightarrow B \subseteq A$.

(2) $(A \cup B)(x) = A(x) \vee B(x) = max(A(x), B(x))$;$(A \cap B)(x) = A(x) \wedge B(x) = min(A(x), B(x))$.

(3) $(AB)(x) = A(x)B(x), A^c(x) = 1 - A(x)$.

Definition 2.2[32][17]Let X be a non empty classic set.The three reorganization in X like $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$ meets the following three points.

(1) $\mu_A \to [0,1]$ indicates that the element of X belongs to the A membership degree.

(2) $\nu_A \to [0,1]$ indicates that the non membership degree.

(3) $0 \leq A(x) + \nu_A(x) \leq 1$.

A is called an IF set on the X.

Related operations of IF sets. Suppose

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\} \in IF(X),$$

$$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle | x \in X\} \in IF(X).$$

$A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x), \forall x \in X$;
$A \cap B$
$= \{\langle x, min\{\mu_A(x), \mu_B(x)\rangle\}, max\{\nu_A(x), \nu_B(x)\}\rangle | x \in X\}$;
$A \cup B$
$= \{\langle x, max\{\mu_A(x), \mu_B(x)\rangle\}, min\{\nu_A(x), \nu_B(x)\}\rangle | x \in X\}$;

$$A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in X\}.$$

Definition 2.3[32] The four tuple $I = (U, AT, V, f)$ are called an IFIS, where

(1) $U = \{x_1, x_2, \cdots, x_n\}$ Where an arbitrary $x_i \in U$ is an object, $i = 1, 2, \cdots, n$;

(2) $AT = \{a_1, a_2, \cdots, a_p\}$ Where an arbitrary $a_j \in U$ is an attributes, $j = 1, 2, \cdots, p$;

(3) $V = \bigcup\limits_{a \in AT} V_a$ and the value of the attribute a is V_a;

(4) $f : U \times AT \Rightarrow V$, there f is called a function such that$f(x, a) \in V_a$,for each $a \in AT, x \in U$, where V_a is called an IF set valued about U. This is $f(x, a) = \langle \mu_a(x), \nu_a(x)\rangle$, for every $a \in AT$.

B. Fuzzy Approximation Space

Definition 2.4[34] Suppose $\mathcal{R}$ is a fuzzy relation, $\mathcal{R}$ has the following three properties

(1) If for any $x \in U$, $\mathcal{R}(x, x) = 1$, $\mathcal{R}$ is called satisfying reflexivity.

(2) If for any $x, y \in U$, $\mathcal{R}(x, y) = \mathcal{R}(y, x)$, $\mathcal{R}$ is is called satisfaction symmetry.

(3) If for any $x, y, z \in U, \mathcal{R}(x, y) \geq \vee_{z \in U}(\mathcal{R}(x, z) \wedge \mathcal{R}(z, y))$, $\mathcal{R}$ is called satisfying transitivity.

On U, if $\mathcal{R}$ satisfies both reflexivity, symmetry and transitivity at the same time, therefor $\mathcal{R}$ is called a fuzzy equivalence relation on U.

If $U = \{x_1, x_2, \cdots, x_n\}$ is a non-empty finite set, $\mathcal{R} : U \times U \to L$ can be represented by an fuzzy matrix form$\mathcal{R} = (\mathcal{R}(x_i, x_j))_{n \times n}$,ie. $\mathcal{R}$ is called fuzzy relation. Then

$$\mathcal{R} = \begin{pmatrix} (\mathcal{R}(x_1, x_1) & \mathcal{R}(x_1, x_2) & \cdots & \mathcal{R}(x_1, x_n)) \\ (\mathcal{R}(x_2, x_1) & \mathcal{R}(x_2, x_2) & \cdots & \mathcal{R}(x_2, x_n)) \\ \vdots & \vdots & \ddots & \vdots \\ (\mathcal{R}(x_n, x_1) & \mathcal{R}(x_n, x_2) & \cdots & \mathcal{R}(x_n, x_n)) \end{pmatrix}.$$

$\mathcal{I} = (U, \mathcal{R})$ is called fuzzy approximation space, which U is the universe, then the $\mathcal{R}$ is an equivalence relation on U .

Definition 2.5[9] Suppose $\mathcal{I} = (U, \mathcal{R})$ is a fuzzy approximation space, U is a classical non-empty and finite set of objects. $\mathcal{R}$ is called fuzzy equivalence relation on U, $A \subseteq U$.

$\underline{\mathcal{R}}_A(y) = \wedge\{1 - \mathcal{R}(x, y) | x \notin A\}$;
$\overline{\mathcal{R}}_A(y) = \vee\{\mathcal{R}(x, y) | x \in A\}$

$\underline{\mathcal{R}}_A$ and $\overline{\mathcal{R}}_A$ are called the fuzzy lower approximation and upper approximation of A in $\mathcal{I}$, respectively. If $\underline{\mathcal{R}}_A \neq \overline{\mathcal{R}}_A$, then A is called rough fuzzy set.

The positive, negative, and boundary region of A are defined as follows:
$POS_{\mathcal{R}}A = \underline{\mathcal{R}}_A, NEG_{\mathcal{R}}A = (\overline{\mathcal{R}}_A)^c, BND_{\mathcal{R}}A = \overline{\mathcal{R}}_A - \underline{\mathcal{R}}_A$.

Proposition 2.1[37]The lower and upper approximation of A have the following properties.

(1) $\underline{\mathcal{R}}_A \subseteq A \subseteq \overline{\mathcal{R}}_A$.
(2) $\underline{\mathcal{R}}_{A^c} = (\overline{\mathcal{R}}_A)^c, \overline{\mathcal{R}}_{A^c} = (\underline{\mathcal{R}}_A)^c$.
(3) $\underline{\mathcal{R}}_\varnothing = \varnothing, \overline{\mathcal{R}}_\varnothing = \varnothing, \underline{\mathcal{R}}_U = U, \overline{\mathcal{R}}_U = U$.
(4) $\underline{\mathcal{R}}_{A \cap B} = \underline{\mathcal{R}}_A \cap \underline{\mathcal{R}}_B, \overline{\mathcal{R}}_{A \cup B} = \overline{\mathcal{R}}_A \cup \overline{\mathcal{R}}_B$.
(5) $A \subseteq B \Rightarrow \underline{\mathcal{R}}_A \subseteq \underline{\mathcal{R}}_B, A \subseteq B \Rightarrow \overline{\mathcal{R}}_A \subseteq \overline{\mathcal{R}}_B$.
(6) $\underline{\mathcal{R}}_A \cup \underline{\mathcal{R}}_B \subseteq \underline{\mathcal{R}}_{A \cup B}, \overline{\mathcal{R}}_{A \cap B} \subseteq \overline{\mathcal{R}}_A \cap \overline{\mathcal{R}}_B$.

Proof:

(1)$\forall y \in \underline{\mathcal{R}}_A \Rightarrow \underline{\mathcal{R}}_A(y) = 1 \Rightarrow \mathcal{R}(x, y) = 0, x \notin A, \Rightarrow y \in A$; $\forall y \in A \Rightarrow \vee\{\mathcal{R}(x, y) | x \in A\} = 1 \Rightarrow \overline{\mathcal{R}}_A(y) = 1 \Rightarrow y \in \overline{\mathcal{R}}_A$. Thus, $\underline{\mathcal{R}}_A \subseteq A \subseteq \overline{\mathcal{R}}_A$.

(2)$\forall y \in \underline{\mathcal{R}}_{A^c} \Leftrightarrow \underline{\mathcal{R}}_{A^c}(y) = \wedge\{1 - \mathcal{R}(x, y) | x \notin A^c\} = 1 \Leftrightarrow y \in A^C \Leftrightarrow \overline{\mathcal{R}}_A(y) = \vee\{\mathcal{R}(x, y) | x \in A\} = 0 \Leftrightarrow y \notin \overline{\mathcal{R}}_A \Leftrightarrow y \in (\overline{\mathcal{R}}_A)^c$. Thus, $\underline{\mathcal{R}}_{A^c} = (\overline{\mathcal{R}}_A)^c$; Similarly, $\overline{\mathcal{R}}_{A^c} = (\underline{\mathcal{R}}_A)^c$.

(3)Obviously set.

(4)$\forall y \in \underline{\mathcal{R}}_{A \cap B} \Leftrightarrow \underline{\mathcal{R}}_{A \cap B}(y) = \wedge\{1 - \mathcal{R}(x, y) | x \notin A \cap B\} = 1 \Leftrightarrow y \in A \cap B \Leftrightarrow \underline{\mathcal{R}}_A(y) = 1 and \underline{\mathcal{R}}_B(y) = 1 \Leftrightarrow y \in \underline{\mathcal{R}}_A \cap \underline{\mathcal{R}}_B$. Thus, $\underline{\mathcal{R}}_{A \cap B} = \underline{\mathcal{R}}_A \cap \underline{\mathcal{R}}_B$; Similarly, $\overline{\mathcal{R}}_{A \cup B} = \overline{\mathcal{R}}_A \cup \overline{\mathcal{R}}_B$.

(5)$\forall y \in \underline{\mathcal{R}}_A \Leftrightarrow \underline{\mathcal{R}}_A(y) = \wedge\{1 - \mathcal{R}(x, y) | x \notin A\} = 1$, because, $A \subseteq B \Rightarrow \wedge\{1 - \mathcal{R}(x, y) | x \notin B\} = 1 \Rightarrow \underline{\mathcal{R}}_B(y) = 1$, Thus, $A \subseteq B \Rightarrow \underline{\mathcal{R}}_A \subseteq \underline{\mathcal{R}}_B$; Similarly, $A \subseteq B \Rightarrow \overline{\mathcal{R}}_A \subseteq \overline{\mathcal{R}}_B$.

(6)By (5) can be directly inferred. ■

III. ROUGH MEMBERSHIP DEGREE BASED ON INTUITIONISTIC FUZZY INFORMATION SYSTEM

Definition 3.1 Suppose $I = (U, AT, V, F)$ is an IFIS, $\forall a_k \in AT$, $x_i, x_j \in U$, $f(x_i, a_k) = \langle \mu_{a_k}(x_i), \nu_{a_k}(x_i)\rangle$, $f(x_j, a_k) = \langle \mu_{a_k}(x_j), \nu_{a_k}(x_j)\rangle$. The similarity degree of objects x_i and x_j under attribute set AT is as follows: $\mathcal{R}(x_i, x_j) = \sum\limits_{k=1}^{p} s_{a_k}(x_i, x_j)/p$.

$s_{a_k}(x_i, x_j)$

$$= \frac{\mu_{a_k}(x_i)\mu_{a_k}(x_j) + \nu_{a_k}(x_i)\nu_{a_k}(x_j)}{\max\{(\mu_{a_k}(x_i))^2 + (\nu_{a_k}(x_i))^2, (\mu_{a_k}(x_j))^2 + (\nu_{a_k}(x_j))^2\}}$$

is called the relative similarity degree of $f(x_i, a_k)$ and $f(x_j, a_k)$. Through establishing analogical relations $\mathcal{R}$, we could turn IFIS into a fuzzy approximation space $(U, \mathcal{R})$ in accordance with definition 2.4. The subscript AT will be omitted in the rear. It holds:

(1) Firstly, U is a non-empty classical set, a binary relation $\mathcal{R}$ from U to U indicates a fuzzy set $\mathcal{R} : U \times U \to [0, 1]$. So $\mathcal{R}$ is a fuzzy relation on the universe U.

(2) Furthermore, $\mathcal{R}$ is a fuzzy equivalence relation on U. The reasons are as follows:
- $\forall x \in U$, $\mathcal{R}(x, x) = 1$, $\mathcal{R}$ is reflexive;
- $\forall x, y \in U$, $\mathcal{R}(x, y) = \mathcal{R}(y, x)$, $\mathcal{R}$ is symmetric;
- $\forall x, y, z \in U$, $\mathcal{R}(x, y) \wedge \mathcal{R}(y, z) \leq \mathcal{R}(x, z)$, $\mathcal{R}$ is transitive.

These three conditions are very obvious. Therefore, ordered pair $(U, \mathcal{R})$ forms a fuzzy approximation space.

Definition 3.2 Suppose $I = (U, AT, V, F)$ is an IFIS, $\mathcal{R}$ is fuzzy equivalence relation of I. $A \subseteq U$, for any $y \in U$, rough membership measure of y in A with respect to $\mathcal{R}$ that is denoted by

$$\mathcal{U}^A_{\mathcal{R}}(y) = \frac{\sum\limits_{x_i \in A} \mathcal{R}(x_i, y)}{\sum\limits_{x_j \in U} \mathcal{R}(x_j, y)}.$$

Apparently, for any $y \in U$, $0 \leq \mathcal{U}^A_{\mathcal{R}}(y) \leq 1$.

Proposition 3.1 Suppose $I = (U, AT, V, F)$ is an IF information system, $\mathcal{R}$ is fuzzy equivalence relation of I. $A \subseteq U$, for any $y \in U$. The properties of rough membership measure are as follows:

(1) $\mathcal{U}^U_{\mathcal{R}}(y) = 1$;
(2) $\mathcal{U}^{\varnothing}_{\mathcal{R}}(y) = 0$;
(3) $\mathcal{U}^A_{\mathcal{R}}(y) = 1 \Leftrightarrow y \in POS_{\mathcal{R}}A$;
(4) $\mathcal{U}^A_{\mathcal{R}}(y) = 0 \Leftrightarrow y \in NEG_{\mathcal{R}}A$;
(5) $0 < \mathcal{U}^A_{\mathcal{R}}(y) < 1 \Leftrightarrow y \in BND_{\mathcal{R}}A$;
(6) $\mathcal{U}^{A^c}_{\mathcal{R}}(y) = 1 - \mathcal{U}^A_{\mathcal{R}}(y)$.

Proof:

(1)On the basis of **Definition 3.2**,when $A = U$,we can easily get

$$\mathcal{U}^U_{\mathcal{R}}(y) = \frac{\sum\limits_{x_i \in U} \mathcal{R}(x_i, y)}{\sum\limits_{x_j \in U} \mathcal{R}(x_j, y)} = 1.$$

(2) Similar to (1) from the **Definition 3.2**, when $A = \varnothing$, for any $x_i \in \varnothing, \mathcal{R}(x_i, y) = 0$. Therefore e can obtain

$$\mathcal{U}^{\varnothing}_{\mathcal{R}}(y) = \frac{\sum\limits_{x_i \in \varnothing} \mathcal{R}(x_i, y)}{\sum\limits_{x_j \in U} \mathcal{R}(x_j, y)} = 0.$$

(3) On the basis of **Definition 3.2** and **Definition 2.5**, we can get:

$\mathcal{U}^A_{\mathcal{R}}(y) = \frac{\sum_{x_i \in \varnothing} \mathcal{R}(x_i,y)}{\sum_{x_j \in U} \mathcal{R}(x_j,y)} = 1 \Leftrightarrow \sum\limits_{x_k \in A^c} \mathcal{R}(x_k, y) = 0 \Leftrightarrow \wedge\{1 - \mathcal{R}(x_k, y)|x_k \in A^c\} = 1 \Leftrightarrow \underline{\mathcal{R}}(y) = 1 \Leftrightarrow y \in \underline{\mathcal{R}} \Leftrightarrow y \in POS_{\mathcal{R}}A$.

(4) Similar to (2) from the **Definition 3.2** and **Definition 2.5**, we can obtain:

$\mathcal{U}^A_{\mathcal{R}}(y) = \frac{\sum_{x_i \in \varnothing} \mathcal{R}(x_i,y)}{\sum_{x_j \in U} \mathcal{R}(x_j,y)} = 0 \Leftrightarrow \sum\limits_{x_i \in A} \mathcal{R}(x_i, y) = 0 \Leftrightarrow \{\vee \mathcal{R}(x_i, y)|x_i \in A = 0\} \Leftrightarrow \overline{\mathcal{R}}(y) = 0 \Leftrightarrow y \notin \overline{\mathcal{R}} \Leftrightarrow y \in (\overline{\mathcal{R}})^c \Leftrightarrow y \in NEG_{\mathcal{R}}A$.

(5) According to (3) and (4), apparently ,the Proposition be derived.

(6) On the basis of **Definition 3.2**, we can easily get

$$\mathcal{U}^{U-A}_{\mathcal{R}}(y) = \frac{\sum\limits_{x_i \in U-A} \mathcal{R}(x_i, y)}{\sum\limits_{x_j \in U} \mathcal{R}(x_j, y)} = \frac{\sum\limits_{x_i \in U} \mathcal{R}(x_i, y) - \sum\limits_{x_i \in A} \mathcal{R}(x_i, y)}{\sum\limits_{x_j \in U} \mathcal{R}(x_j, y)}$$

$= 1 - \mathcal{U}^A_{\mathcal{R}}(y)$. ■

Proposition 3.2[26] Suppose $I = (U, AT, V, F)$ be an IFIS, $\mathcal{R}$ is fuzzy equivalence relation of I. $A \subseteq U$, for any $y \in U$, and mainly found the following conclusions:

(1) $\mathcal{U}^A_{\mathcal{R}}(y) = 1 \Rightarrow y \in A$;
(2) $\mathcal{U}^A_{\mathcal{R}}(y) = 0 \Rightarrow y \notin A$;

Proof:

(1) On the basis of **Proposition 3.1**(3),we can obtain: $\mathcal{U}^A_{\mathcal{R}}(y) = 1 \Rightarrow y \in POS_{\mathcal{R}}A \Rightarrow y \in \underline{\mathcal{R}}_A \Rightarrow \mathcal{R}_A(y) = 1 \Rightarrow \wedge\{1 - \mathcal{R}(x_i, y)|x_i \in A^c\} = 1 \Rightarrow \{\vee \mathcal{R}(x_i, y)|x_i \in A^c\} = 0 \Rightarrow \forall x_i \in A^c, \mathcal{R}(x_i, y) = 0 \Rightarrow y \notin A^c \Rightarrow y \in A$.

(2) On the basis of **Proposition 3.1**(4),we can obtain: $\mathcal{U}^A_{\mathcal{R}}(y) = 0 \Rightarrow y \in NEG_{\mathcal{R}}A \Rightarrow y \in (\overline{\mathcal{R}}_A)^c \Rightarrow \overline{\mathcal{R}}_A(y) = 0 \Rightarrow \{\vee \mathcal{R}(x_i, y)|x_i \in A^c\} = 0 \Rightarrow \forall x_i \in A, \mathcal{R}(x_i, y) = 0 \Rightarrow y \notin A \Rightarrow y \in A^c$. ■

Remark

(1) $y \in A \nRightarrow \mathcal{U}^A_{\mathcal{R}}(y) = 1$;
(2) $y \notin A \nRightarrow \mathcal{U}^A_{\mathcal{R}}(y) = 0$;

Proof: Let $U = \{x_1, x_2, x_3\}$, $A = \{x_1, x_3\}$.

A fuzzy relation on U: $\mathcal{R}(x_1, x_1) = 1$, $\mathcal{R}(x_1, x_2) = 0.40$, $\mathcal{R}(x_1, x_3) = 0.60$, $\mathcal{R}(x_2, x_1) = 0.40$, $\mathcal{R}(x_2, x_2) = 1$, $\mathcal{R}(x_2, x_3) = 0.80$, $\mathcal{R}(x_3, x_1) = 0.60$, $\mathcal{R}(x_3, x_2) = 0.80$, $\mathcal{R}(x_3, x_3) = 1$.

(1)when $y = x_1, y \in A$, $\mathcal{U}^A_{\mathcal{R}}(y) = 0.8 \neq 1$, Thus, $y \in A \nRightarrow \mathcal{U}^A_{\mathcal{R}}(y) = 1$.

(2)when $y = x_2, y \notin A$, $\mathcal{U}^A_{\mathcal{R}}(y) = 0.55 \neq 0$, Thus, $y \notin A \nRightarrow \mathcal{U}^A_{\mathcal{R}}(y) = 0$. ■

Proposition 3.3 Suppose $I = (U, AT, V, F)$ is an IF information system, $\mathcal{R}$ is fuzzy equivalence relation of I. $A \subseteq U$, for any $y \in U$, for any $A, B \subseteq U$, and therefor concludes as follows:

(1) $A \subseteq B \Rightarrow \mathcal{U}^A_{\mathcal{R}}(y) \leq \mathcal{U}^B_{\mathcal{R}}(y)$;
(2) $\mathcal{U}^{A \cup B}_{\mathcal{R}}(y) \geq \vee\{\mathcal{U}^A_{\mathcal{R}}(y), \mathcal{U}^B_{\mathcal{R}}(y)\}$;
(3) $\mathcal{U}^{A \cap B}_{\mathcal{R}}(y) \leq \wedge\{\mathcal{U}^A_{\mathcal{R}}(y), \mathcal{U}^B_{\mathcal{R}}(y)\}$;
(4) $\mathcal{U}^{A \cup B}_{\mathcal{R}}(y) = \mathcal{U}^A_{\mathcal{R}}(y) + \mathcal{U}^B_{\mathcal{R}}(y) - \mathcal{U}^{A \cap B}_{\mathcal{R}}(y)$;
(5) $A \cap B = \varnothing \Rightarrow \mathcal{U}^{A \cup B}_{\mathcal{R}}(y) = \mathcal{U}^A_{\mathcal{R}}(y) + \mathcal{U}^B_{\mathcal{R}}(y)$.

Proof:

(1) when $A \subseteq B$
$\mathcal{U}_{\mathcal{R}}^{B}(y) - \mathcal{U}_{\mathcal{R}}^{A}(y)$

$$= \frac{\sum\limits_{x_i \in B} \mathcal{R}(x_i, y) - \sum\limits_{x_k \in A} \mathcal{R}(x_k, y)}{\sum\limits_{x_j \in U} \mathcal{R}(x_j, y)} = \frac{\sum\limits_{x_r \in (B-A)} \mathcal{R}(x_r, y)}{\sum\limits_{x_j \in U} \mathcal{R}(x_j, y)} \geq 0$$

Thus, $\mathcal{U}_{\mathcal{R}}^{A}(y) \leq \mathcal{U}_{\mathcal{R}}^{B}(y)$.
(2) $\mathcal{U}_{\mathcal{R}}^{A\cup B}(y) - \mathcal{U}_{\mathcal{R}}^{A}(y)$

$$= \frac{\sum\limits_{x_i \in A\cup B} \mathcal{R}(x_i, y) - \sum\limits_{x_k \in A} \mathcal{R}(x_k, y)}{\sum\limits_{x_j \in U} \mathcal{R}(x_j, y)} = \frac{\sum\limits_{x_r \in B} \mathcal{R}(x_r, y)}{\sum\limits_{x_j \in U} \mathcal{R}(x_j, y)} \geq 0$$

$\Leftrightarrow \mathcal{U}_{\mathcal{R}}^{A\cup B}(y) \leq \mathcal{U}_{\mathcal{R}}^{A}(y)$.
Similarly, $\mathcal{U}_{\mathcal{R}}^{A\cup B}(y) \leq \mathcal{U}_{\mathcal{R}}^{A}(y)$.
Thus, $\mathcal{U}_{\mathcal{R}}^{A\cup B}(y) \geq \vee\{\mathcal{U}_{\mathcal{R}}^{A}(y), \mathcal{U}_{\mathcal{R}}^{B}(y)\}$.
(3) $\mathcal{U}_{\mathcal{R}}^{A}(y) - \mathcal{U}_{\mathcal{R}}^{A\cap B}(y)$

$$= \frac{\sum\limits_{x_i \in A} \mathcal{R}(x_i, y) - \sum\limits_{x_k \in A\cap B} \mathcal{R}(x_k, y)}{\sum\limits_{x_j \in U} \mathcal{R}(x_j, y)} = \frac{\sum\limits_{x_r \in (A-(A\cap B))} \mathcal{R}(x_r, y)}{\sum\limits_{x_j \in U} \mathcal{R}(x_j, y)}$$

$\geq 0 \Leftrightarrow \mathcal{U}_{\mathcal{R}}^{A}(y) \geq \mathcal{U}_{\mathcal{R}}^{A\cap B}(y)$.
Similarly, $\mathcal{U}_{\mathcal{R}}^{B}(y) \geq \mathcal{U}_{\mathcal{R}}^{A\cap B}(y)$.
Thus, $\mathcal{U}_{\mathcal{R}}^{A\cap B}(y) \leq \wedge\{\mathcal{U}_{\mathcal{R}}^{A}(y), \mathcal{U}_{\mathcal{R}}^{B}(y)\}$.

(4) $\mathcal{U}_{\mathcal{R}}^{A\cup B}(y) = \frac{\sum\limits_{x_i \in (A\cup B)} \mathcal{R}(x_i, y)}{\sum\limits_{x_j \in U} \mathcal{R}(x_j, y)}$

$$= \frac{\sum\limits_{x_m \in A} \mathcal{R}(x_m, y) + \sum\limits_{x_n \in B} \mathcal{R}(x_n, y) - \sum\limits_{x_k \in (A\cap B)} \mathcal{R}(x_k, y)}{\sum\limits_{x_j \in U} \mathcal{R}(x_j, y)}$$

$= \mathcal{U}_{\mathcal{R}}^{A}(y) + \mathcal{U}_{\mathcal{R}}^{B}(y) - \mathcal{U}_{\mathcal{R}}^{A\cap B}(y)$.
(5) On the basis of proposition(4)

$$\mathcal{U}_{\mathcal{R}}^{A\cup B}(y) = \mathcal{U}_{\mathcal{R}}^{A}(y) + \mathcal{U}_{\mathcal{R}}^{B}(y) - \mathcal{U}_{\mathcal{R}}^{A\cap B}(y).$$

According to **Proposition 3.1**(2), $A \cap B = \varnothing \Rightarrow \mathcal{U}_{\mathcal{R}}^{A\cap B}(y) = 0$.

Thus, $A \cap B = \varnothing \Rightarrow \mathcal{U}_{\mathcal{R}}^{A\cup B}(y) = \mathcal{U}_{\mathcal{R}}^{A}(y) + \mathcal{U}_{\mathcal{R}}^{B}(y)$. ∎

Proposition 3.4 Suppose $I = (U, AT, V, F)$ is an IF information system, $\mathcal{R}$ is fuzzy equivalence relation of I. $A \subseteq U$, for any $y \in U$. $A = \{A_1, A_2, \ldots, A_N\}$ is a family of sets, $\forall A_i, A_j \in A, A_i \cap A_j = \varnothing$, and therefor concludes as follows:

(1) $\mathcal{U}_{\mathcal{R}}^{\bigcup A_i}(y) = \sum\limits_{A_i \in A} \mathcal{U}_{\mathcal{R}}^{A_i}(y)$;

(2) If $U = A_1 \cup A_2 \cup \cdots \cup A_N$ and $A_i \cap A_j = \emptyset$, then $\mathcal{U}_{\mathcal{R}}^{\bigcup A_i}(y) = 1$.

Proof:

(1) From the (4) and (5) in the **Proposition 3.3**, we can obtain $\mathcal{U}_{\mathcal{R}}^{\bigcup A_i}(y) = \sum\limits_{A_i \in A} \mathcal{U}_{\mathcal{R}}^{A_i}(y)$.

(2)According to the known conditions, we can obtain $\bigcup A_i = U$. So we can get the following equation:

$$\mathcal{U}_{\mathcal{R}}^{\bigcup A_i}(y) = \mathcal{U}_{\mathcal{R}}^{A_1}(y) + \mathcal{U}_{\mathcal{R}}^{A_2}(y) + \cdots + \mathcal{U}_{\mathcal{R}}^{A_N}(y) =$$

$$\frac{\sum\limits_{x_{i_1} \in A_1} \mathcal{R}(x_{i_1}, y) + \sum\limits_{x_{i_2} \in A_2} \mathcal{R}(x_{i_2}, y) + \cdots + \sum\limits_{x_{i_N} \in A_N} \mathcal{R}(x_{i_N}, y)}{\sum\limits_{x_j \in U} \mathcal{R}(x_j, y)}$$

$$= \frac{\sum\limits_{x_i \in \bigcup A_i} \mathcal{R}(x_i, y)}{\sum\limits_{x_j \in U} \mathcal{R}(x_j, y)} = \frac{\sum\limits_{x_i \in U} \mathcal{R}(x_i, y)}{\sum\limits_{x_j \in U} \mathcal{R}(x_j, y)} = 1.$$

∎

TABLE I
The intuitionistic fuzzy information system

U	a_1	a_2	a_3	a_4	a_5
x_1	$\langle 0.4, 0.4\rangle$	$\langle 0.3, 0.5\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.4, 0.5\rangle$	$\langle 0.7, 0.1\rangle$
x_2	$\langle 0.3, 0.5\rangle$	$\langle 0.2, 0.5\rangle$	$\langle 0.6, 0.1\rangle$	$\langle 0.4, 0.5\rangle$	$\langle 0.7, 0.3\rangle$
x_3	$\langle 0.3, 0.5\rangle$	$\langle 0.4, 0.4\rangle$	$\langle 0.6, 0.1\rangle$	$\langle 0.4, 0.5\rangle$	$\langle 0.7, 0.3\rangle$
x_4	$\langle 0.8, 0.1\rangle$	$\langle 0.8, 0.1\rangle$	$\langle 0.5, 0.4\rangle$	$\langle 0.8, 0.1\rangle$	$\langle 0.8, 0.2\rangle$
x_5	$\langle 0.5, 0.3\rangle$	$\langle 0.4, 0.5\rangle$	$\langle 0.7, 0.1\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.8, 0.1\rangle$
x_6	$\langle 0.3, 0.3\rangle$	$\langle 0.2, 0.3\rangle$	$\langle 0.4, 0.2\rangle$	$\langle 0.5, 0.1\rangle$	$\langle 0.0, 0.7\rangle$
x_7	$\langle 0.4, 0.6\rangle$	$\langle 0.5, 0.1\rangle$	$\langle 0.6, 0.2\rangle$	$\langle 0.2, 0.6\rangle$	$\langle 0.3, 0.6\rangle$
x_8	$\langle 0.6, 0.1\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.4, 0.2\rangle$	$\langle 0.5, 0.3\rangle$	$\langle 0.7, 0.1\rangle$
x_9	$\langle 0.6, 0.3\rangle$	$\langle 0.4, 0.0\rangle$	$\langle 0.6, 0.1\rangle$	$\langle 0.8, 0.2\rangle$	$\langle 0.2, 0.0\rangle$
x_{10}	$\langle 0.0, 1.0\rangle$	$\langle 0.3, 0.5\rangle$	$\langle 0.2, 0.6\rangle$	$\langle 0.6, 0.2\rangle$	$\langle 0.0, 0.2\rangle$

Example 3.1 As shown in TABLE, $I = (U, AT, V, F)$ is an IFIS. Among $AT = \{a_1, a_2, a_3, a_4, a_5\}, U = \{x_1, x_2, \cdots, x_{10}\}$, $\mathcal{R}$ is fuzzy equivalence relation of I, $\mathcal{I} = (U, \mathcal{R})$ is a fuzzy approximation space on I.

On the basis of **Definition 3.1** we can get fuzzy relation $\mathcal{R}$ in TABLE.

TABLE II
A fuzzy relation on U

U	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
x_1	1.00									
x_2	0.92	1.00								
x_3	0.92	0.98	1.00							
x_4	0.65	0.62	0.66	1.00						
x_5	0.92	0.87	0.89	0.73	1.00					
x_6	0.54	0.61	0.60	0.45	0.51	1.00				
x_7	0.74	0.77	0.82	0.60	0.70	0.61	1.00			
x_8	0.76	0.71	0.75	0.77	0.78	0.58	0.64	1.00		
x_9	0.58	0.57	0.62	0.66	0.62	0.48	0.61	0.61	1.00	
x_{10}	0.55	0.56	0.56	0.44	0.49	0.50	0.53	0.43	0.37	1.00

Let $A = \{A_1, A_2, A_3\}$ is a partition of U, among $A_1 = \{x_1, x_3, x_6\}, A_2 = \{x_2, x_4, x_5\}, A_3 = \{x_7, x_8, x_9, x_{10}\}$

According to **Definition 3.2** we can get the rough membership measure of $x_i (x_i \in U)$ in A_1, A_2 and A_3 with respect to $\mathcal{R}$, and therefor results as follows:

$$\mathcal{U}_{\mathcal{R}}^{A_1}(x_1) = 0.326, \mathcal{U}_{\mathcal{R}}^{A_2}(x_1) = 0.327, \mathcal{U}_{\mathcal{R}}^{A_3}(x_1) = 0.347$$

$$\mathcal{U}_{\mathcal{R}}^{A_1}(x_2) = 0.329, \mathcal{U}_{\mathcal{R}}^{A_2}(x_2) = 0.327, \mathcal{U}_{\mathcal{R}}^{A_3}(x_2) = 0.344$$

$$\mathcal{U}_{\mathcal{R}}^{A_1}(x_3) = 0.324, \mathcal{U}_{\mathcal{R}}^{A_2}(x_3) = 0.323, \mathcal{U}_{\mathcal{R}}^{A_3}(x_3) = 0.353$$

$$\mathcal{U}_{\mathcal{R}}^{A_1}(x_4) = 0.267, \mathcal{U}_{\mathcal{R}}^{A_2}(x_4) = 0.357, \mathcal{U}_{\mathcal{R}}^{A_3}(x_4) = 0.376$$

$$\mathcal{U}_{\mathcal{R}}^{A_1}(x_5) = 0.309, \mathcal{U}_{\mathcal{R}}^{A_2}(x_5) = 0.347, \mathcal{U}_{\mathcal{R}}^{A_3}(x_5) = 0.344$$

$$\mathcal{U}_{\mathcal{R}}^{A_1}(x_6) = 0.365, \mathcal{U}_{\mathcal{R}}^{A_2}(x_6) = 0.266, \mathcal{U}_{\mathcal{R}}^{A_3}(x_6) = 0.369$$

$$\mathcal{U}_{\mathcal{R}}^{A_1}(x_7) = 0.308, \mathcal{U}_{\mathcal{R}}^{A_2}(x_7) = 0.295, \mathcal{U}_{\mathcal{R}}^{A_3}(x_7) = 0.397$$
$$\mathcal{U}_{\mathcal{R}}^{A_1}(x_8) = 0.298, \mathcal{U}_{\mathcal{R}}^{A_2}(x_8) = 0.321, \mathcal{U}_{\mathcal{R}}^{A_3}(x_8) = 0.381$$
$$\mathcal{U}_{\mathcal{R}}^{A_1}(x_9) = 0.274, \mathcal{U}_{\mathcal{R}}^{A_2}(x_9) = 0.302, \mathcal{U}_{\mathcal{R}}^{A_3}(x_9) = 0.424$$
$$\mathcal{U}_{\mathcal{R}}^{A_1}(x_{10}) = 0.298, \mathcal{U}_{\mathcal{R}}^{A_2}(x_{10}) = 0.273, \mathcal{U}_{\mathcal{R}}^{A_3}(x_{10}) = 0.429$$

Apparently $\{A_1, A_2, A_3\}$ forms a partition of U, through this example we can verify that for any $x_i (x_i \in U)$, $\mathcal{U}_{\mathcal{R}}^{A_1}(x_i) + \mathcal{U}_{\mathcal{R}}^{A_2}(x_i) + \mathcal{U}_{\mathcal{R}}^{A_3}(x_i) = 1$. Thus, **Proposition 3.4** is verified.

IV. Conclusion

In the field of the development and application of rough set theory, extending the classical rough set theory to fuzzy relation front is an important direction. In this paper, we defined the fuzzy equivalence relation in IF information system. Thus, we can obtain fuzzy approximation space. Lower and upper fuzzy approximation of classical set is discussed. The rough memberships measure is defined by fuzzy equivalence relation. Therefor, a few important properties of rough memberships measure have been presented and demonstrated. In further research, we will study the rough memberships measure in IF approximation space.

Acknowledgment

This work is supported by Natural Science Foundation of China (No. 61105041, No. 61472463 and No. 61402064), Macau Science and Technology Development Fundation (No: 081/2015/A3), National Natural Science Foundation of CQ C-STC (No. cstc2015jcyjA40053), Graduate Innovation Foundation of CQ (No. CYS17281), Graduate Innovation Foundation of Chongqing University of Technology (No. YCX2016227).

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